Topic: Precise definition of the limit, finding delta

Question: Given $\lim_{x \to 4} 5x - 3 = 17$, find $\delta > 0$ such that $\left|(5x - 3) - 17\right| < 0.001$ whenever $0 < \left|x - 4\right| < \delta$.

Answer choices:

A 0.2

B 0.02

C 0.002

D 0.0002

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Solution: D

We want to have $\left|(5x-3)-17\right| < 0.001$ whenever $0 < \left|x-4\right| < \delta$. Since

$$\left|(5x-3)-17\right| = \left|5x-20\right| = 5\left|x-4\right| < 0.001$$

implies

$$\left|x-4\right| < \frac{1}{5}(0.001)$$

we can choose $\delta = \frac{1}{5}(0.001) = 0.0002$.

We can see that this choice works by double-checking: whenever $0 < \left|x-4\right| < 0.0002$,

$$\left|(5x-3)-17\right| = \left|5x-20\right| = 5\left|x-4\right| < 5(0.0002) = 0.001$$

Of course, any value smaller than 0.0002 (and greater than 0), will also work for δ.

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Topic: Precise definition of the limit, finding delta

Question: What is δ for $f(x) = x^2$ such that if $|x - 2| < \delta$ then $|x^2 - 4| < 0.5$?

Answer choices:

A $\delta = 1.87$

B $\delta = 0.5$

C $\delta = 0.12$

D $\delta = 2.12$

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Solution: C

Use the value for ϵ to determine the δ value by remembering that

$$0 < |x - a| < \delta \text{ then } |f(x) - L| < \epsilon$$

from the precise definition of the limit.

To solve, take the epsilon ϵ value (0.5) and the value of L to find the two y-values. This means we have $4 + 0.5 = 4.5$ and $4 - 0.5 = 3.5$. Then we can plug these values into the function to get the associated x-values.

$$4.5 = x^2$$

$$x = 2.12$$

and

$$3.5 = x^2$$

$$x = 1.87$$

We'll find $|x - a|$ with these two x-values and $a = 2$.

For $x = 2.12$: $|x - a| = |2.12 - 2| = |0.12| = 0.12$

For $x = 1.87$: $|x - a| = |1.87 - 2| = |-0.13| = 0.13$

If the two values are different, the smaller value will be the value we need to pick for δ. Which means that for the function $f(x) = x^2$ such that if $|x - 2| < \delta$ then $|x^2 - 4| < 0.5$, we know that $\delta = 0.12$.

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Topic: Precise definition of the limit, finding delta

Question: What is δ for $f(x) = (x-1)^2 + 2$ such that if $|x-3| < \delta$ then $|(x-1)^2 + 2 - 6| < 0.2$?

Answer choices:

A $\delta = 0.5$

B $\delta = 0.05$

C $\delta = 1.5$

D $\delta = 1.05$

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Solution: B

Use the value for ϵ to determine the δ value by remembering that

$$0 < |x - a| < \delta \text{ then } |f(x) - L| < \epsilon$$

from the precise definition of the limit.

To solve, take the epsilon ϵ value (0.2) and the value of L to find the two y-values. This means we have $6 + 0.2 = 6.2$ and $6 - 0.2 = 5.8$. Then we can plug these values into the function to get the associated x-values.

$$6.2 = (x-1)^2 + 2$$

$$4.2 = (x-1)^2$$

$$2.05 = x - 1$$

$$x = 3.05$$

and

$$5.8 = (x-1)^2 + 2$$

$$3.8 = (x-1)^2$$

$$1.95 = x - 1$$

$$x = 2.95$$

We'll find $|x - a|$ with these two x-values and $a = 3$.

For $x = 3.05$: $|x - a| = |3.05 - 3| = |0.05| = 0.05$

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For $x = 2.95$: $|x - a| = |2.95 - 3| = |-0.05| = 0.05$

If the two values are different, the smaller value will be the value we need to pick for δ. Which means that for the function $f(x) = (x-1)^2 + 2$ such that if $|x - 3| < \delta$ then $|(x-1)^2 + 2 - 6| < 0.2$, we know that $\delta = 0.05$.

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